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The ENCOMPASSING PRINCIPLE and SPECIFICATION TESTS

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Abstract

The encompassing principle, as a generator of test statistics, is compared to the M–test or conditional moment testing principle. It is shown that the two principles are capable of generating the same test statistics, and in this sense equivalent. However, there are differences in motivation and emphasis underlying the principles which are in important in econometric modelling. The equivalence between encompassing and parsimonious encompassing for nonlinear models is also established for a class of completing models.

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1. Introduction

A major objective in econometric modelling is the development of models which, in addition to having a firm *a priori* theory basis and interpretation, are statistically well specified. The dangers of using models that, although well grounded in economic theory, are statistically incoherent with the available information, have been described and illustrated in *inter alia* Hendry (1985), Mizon (1989) and Spanos (1989). Many strategies for the modelling of relationships between economic variables have been advocated (see the contributions to Granger (1990), and the reviews by Pagan (1988, 1989)), and although, as argued by Hendry and Mizon (1990), there can be no uniquely best modelling strategy, one common feature of most modelling strategies is the emphasis on rigorous model evaluation. For example, Hendry and Mizon (1990) argue for seeking models that are **congruent** with the available information, necessary conditions for which include innovation disturbances, constant parameters, valid conditioning (weak exogeneity) of unmodelled variables for the parameters of interest, consistency with *a priori* theory, and data admissibility, and which **encompass** alternative models of the same phenomena. Each of these requirements for model adequacy can be evaluated using one of the test statistics that have become familiar to econometricians. Indeed, the increasing interest in the use of, and the proliferation of alternative forms for, test statistics has resulted in a vast literature. Examples of contributions to this literature in econometrics include: the specification and/or misspecification test statistics of Chow (1960), Durbin (1970), Engle (1982a), Hausman (1978), Ramsey (1969), Sargan (1980), White (1981, 1982), and Wu (1973). Hendry and Richard (1982, 1983) discuss the role of hypothesis tests in this context, and Engle (1984) provides a review of many of the principles underlying test statistics, particularly the Wald, likelihood ratio, and the Lagrange multiplier principles. In fact, the advantages to having clearly identified principles underlying the multifarious test statistics were evident, and Breusch and Pagan (1980), Engle (1982b), and more recently Godfrey (1988), have demonstrated the important role that the Lagrange multiplier principle has in this regard. Pagan (1984) used the variable addition principle to unify a large number of diagnostic test statistics. Newey (1985) and Tauchen (1985) provided a further step in this direction when they proposed the conditional moment class of test statistics. The principle on which the attractions of embracing virtually all known test statistics, and providing a readily available framework for generating new ones as they are required.

In proposing a statistical framework within which to implement tests of encompassing hypotheses, Mizon (1984) and Mizon and Richard (1986) noted that the Wald encompassing test statistic can be used as a test statistic generator, and that the encompassing principle unifies the literatures on testing nested and non–nested hypotheses, which often previously were regarded as distinct. Hence as a by–product of developing the encompassing principle as an important part of an overall modelling strategy, another means of generating and interpreting test statistics was provided. In fact, as generators of test statistics, which as a result unify an enormous literature on testing, the M–test and the Wald encompassing test (WET) statistics have much in common. In analyzing in more detail the precise relationship between these principles, White (1989) showed that any WET can be obtained as an M–test, and so argued that encompassing tests are special cases of M–tests. In this paper we extend White's results to show that as generators of test statistics with known distributions under the null hypothesis, the M–test principle and the encompassing principle are equivalent. However, the two principles have different motivations and emphases, which can have important implications for the properties of the generated test statistics, and are likely to influence the type of modelling strategy that they are incorporated in.

In the next section the underlying statistical framework for the analysis is described, and this is followed by a presentation of the basic ideas of the conditional moment tests (M–tests), and the encompassing tests ($\mathcal{E}_p$ and $\mathcal{E}$ tests). Section 3 contains analysis of the relationship between M–tests and $\mathcal{E}$–tests, and section 4 shows an equivalence between encompassing ($\mathcal{E}$) comparison of non–nested nonlinear models, and parsimonious encompassing ($\mathcal{E}_p$) with respect to a completing model derived from the exponentially extended family of densities (see Barndorff–Nielsen and Cox (1989)). When the exponential extension incorporates the log–likelihood ratio, this latter equivalence is shown, in section 5, to provide a parsimonious encompassing interpretation of the Cox (1961, 1962) generalized likelihood ratio test. Further, in this case the exponentially extended density is that proposed by Atkinson (1970) for comparing two non–nested models, and so provides an encompassing interpretation of Atkinson's (1970) test. Section 5 ends with an illustration of these results in the case that M_1 and M_2 are linear regression models. Section 6 contains a summary and conclusions.

2. Statistical Framework

We assume that observations are available on a n–dimensional vector of random variables $\mathbf{x}_t$, and that the data generation process (DGP) for these variables satisfies the following conditions.

Assumption 1: The $n \times 1$ random vector $\mathbf{x}_t$ is defined on a Euclidean measurable space $(\Omega, \mathscr{F}, \nu)$ with Ω chosen to be $\mathbb{R}^n$, $\mathscr{F}$ is a Borel σ–algebra of subsets of Ω, and ν is Lebesgue measure.

Assumption 2: The random sequence $\{\mathbf{x}_t, \ t=1,2,......\}$ consists of identically distributed random vectors with joint distribution function $H(\mathbf{x})$ on $(\Omega, \mathscr{F}, \nu)$. The measurable Radon–Nikodym density $h(\mathbf{x})$ exists and is unique up to set of measure zero in ν, such that

$$h(\mathbf{x}) = dH(\mathbf{x})/d\nu.$$

To facilitate the discussion of conditional distributions $\mathbf{x}_t$ is partitioned into $(\mathbf{y}_t{'}, \mathbf{z}_t{'})$, where $\mathbf{y}_t$ and $\mathbf{z}_t$ are $p \times 1$ and $q \times 1$ respectively. Letting $\mathbf{w}_t$ include the conditioning variables $\mathbf{z}_t$ and some lagged variables $\mathbf{x}_{t-j}$ $(j > 0)$, and $\mathscr{F}_t(\mathbf{w}_t, \mathbf{y}_t)$ be a σ–algebra in $\mathscr{F}$, the distribution of $\mathbf{y}_t$ conditional on $\mathbf{w}_t$ is given by $H(\mathbf{y}_t \mid \mathbf{w}_t;\boldsymbol{\alpha})$, where $\boldsymbol{\alpha} \in A$ a compact subset in $\mathbb{R}^m$.

Assumption 3: The conditional distribution of $\mathbf{y}_t$ given $\mathbf{w}_t$, $H(\mathbf{y}_t | \mathbf{w}_t;\boldsymbol{\alpha})$, is defined on $(\Omega_y, \mathscr{F}_y, \nu_y)$, the Euclidean measurable space related to $\mathbf{y}_t$, such that for each $\boldsymbol{\alpha} \in A$, $H(\mathbf{y}_t | \mathbf{w}_t;\boldsymbol{\alpha})$ is $\mathscr{F}_y$ measurable and continuous on A for each $(\mathbf{w}_t, \mathbf{y}_t)$ almost surely.

Assumption 4: The measurable Radon–Nikodym density

$$h(\mathbf{y}_t | \mathbf{w}_t;\boldsymbol{\alpha}) = dH(\mathbf{y}_t | \mathbf{w}_t;\boldsymbol{\alpha})/d\nu_y$$

exists and is unique up to sets of measure zero on ν_y.

Assumptions 1–4 provide the basic statistical framework for the assumed data generation process (DGP). The inclusion of lagged $\mathbf{x}_t$ in $\mathbf{w}_t$ makes it explicit that the framework is general enough to allow the evaluation of dynamic econometric models such as single equation autoregressive–distributed lag models, dynamic simultaneous equations models, and when $\mathbf{w}_t$ consists only of lagged $\mathbf{x}_t$, vector autoregressive models (VAR's). Although $\mathbf{w}_t$ intuitively can be considered as explanatory variables, for it to be legitimate to base inference solely on the conditional distribution $H(\mathbf{y}_t | \mathbf{w}_t;\boldsymbol{\alpha})$ it is necessary that $\mathbf{w}_t$ be weakly exogenous for the parameters of interest φ say (see Engle *et al* (1983)).

It is now necessary to specify the class of statistical models that econometricians might consider in the process of determining a congruent statistical framework within which to develop and test particular econometric models (see e.g. Hendry (1985), Hendry and Mizon (1990), and Spanos (1986, 1990) for general discussions, and Hendry and Mizon (1992) for such analysis within the framework of VAR models), and for which the subsequent analysis will apply. In doing this it is convenient to consider more than one class of statistical distribution so that encompassing comparisons of nonnested or separate densities can be analyzed explicitly.

Assumption 5:

(i) Two statistical models are given by the families of distribution functions:

$$M_1: \{ F(\mathbf{y}_t | \mathbf{w}_t; \boldsymbol{\theta}): \ \boldsymbol{\theta} \in \Xi \subset \mathbb{R}^{\ell} \}$$

and

$$M_2: \{ G(\mathbf{y}_t | \mathbf{w}_t; \boldsymbol{\gamma}): \ \boldsymbol{\gamma} \in \Gamma \subset \mathbb{R}^{S} \}$$

where Ξ and Γ are the respective compact parameter spaces.

(ii) $F(\mathbf{y}_t | \mathbf{w}_t; \boldsymbol{\theta})$ and $G(\mathbf{y}_t | \mathbf{w}_t; \boldsymbol{\gamma})$ are measurable for each $\boldsymbol{\theta} \in \Xi$ and $\boldsymbol{\gamma} \in \Gamma$, and are continuous on Ξ and Γ for each $(\mathbf{y}_t, \mathbf{w}_t)$. The Radon–Nikodym densities:

$$f(\mathbf{y}_t | \mathbf{w}_t; \boldsymbol{\theta}) = dF(\mathbf{y}_t | \mathbf{w}_t; \boldsymbol{\theta}) / d\nu_y$$

and

$$g(\mathbf{y}_t | \mathbf{w}_t; \boldsymbol{\gamma}) = dG(\mathbf{y}_t | \mathbf{w}_t; \boldsymbol{\gamma}) / d\nu_y$$

exist.

(iii) $|\log f(\mathbf{y}_t | \mathbf{w}_t; \boldsymbol{\theta})|$ and $|\log g(\mathbf{y}_t | \mathbf{w}_t; \boldsymbol{\gamma})|$ are dominated by integrable functions with respect to $H(\mathbf{x})$.

(iv) $|\log g(\mathbf{y}_t | \mathbf{w}_t; \boldsymbol{\gamma})|$ is dominated by an integrable function with respect to $F(\mathbf{x})$.

Assumptions 5 (iii) and (iv) ensure the existence of the following expectations:

$$E_w E_0 \log f(\mathbf{y}_t | \mathbf{w}_t; \boldsymbol{\theta}) = E_w \int \log f(\mathbf{y}_t | \mathbf{w}_t; \boldsymbol{\theta}) \, h(\mathbf{y}_t | \mathbf{w}_t; \boldsymbol{\alpha}) \, d\nu_y$$
$$E_w E_0 \log g(\mathbf{y}_t | \mathbf{w}_t; \boldsymbol{\gamma}) = E_w \int \log g(\mathbf{y}_t | \mathbf{w}_t; \boldsymbol{\gamma}) \, h(\mathbf{y}_t | \mathbf{w}_t; \boldsymbol{\alpha}) \, d\nu_y$$
$$E_w E_1 \log g(\mathbf{y}_t | \mathbf{w}_t; \boldsymbol{\gamma}) = E_w \int \log g(\mathbf{y}_t | \mathbf{w}_t; \boldsymbol{\gamma}) \, f(\mathbf{y}_t | \mathbf{w}_t; \boldsymbol{\theta}) \, d\nu_y$$

where E_0 and E_1 denote the expectation operator under the conditional distributions $H(\mathbf{y}_t | \mathbf{w}_t; \boldsymbol{\alpha})$ and $F(\mathbf{y}_t | \mathbf{w}_t; \boldsymbol{\theta})$ respectively, and E_w denotes the expectation under the marginal distribution of $\mathbf{w}_t$. Note that although two separate classes of model are considered, it is essential, in order to compare models from each class (either via $\mathscr{E}$ or M–testing), that they be defined with respect to a common probability distribution. Clearly, $H(\mathbf{x})$ is one such distribution function, but in practice modelling will be supported by a class of completing models M_c which has M_1 and M_2 as special cases. The choice of M_c is extremely important: in particular, (i) it must be sufficiently general to embed or nest all models M_i (i = 1,2,3,.....) which will be entertained as alternative models of the distribution of $\mathbf{y}_t | \mathbf{w}_t$; and (ii) it should be congruent, and thus have fully exploited the available information. These requirements of M_c are to ensure that it has two properties in common with the DGP, $H(\mathbf{x})$.

Noting further that the DGP for $\mathbf{x}_t$ is likely to be highly complicated (nonlinear, dynamic and high dimensional), and that relatively small samples of data are usually available, it is to be expected that $F(\mathbf{y}_t | \mathbf{w}_t; \boldsymbol{\theta})$ and $G(\mathbf{y}_t | \mathbf{w}_t; \boldsymbol{\gamma})$ will be derivable via transformations, conditioning, and marginalization from $H(\mathbf{x})$. Accordingly, $H(\mathbf{y}_t | \mathbf{w}_t; \boldsymbol{\alpha}) \ \mathscr{E} \ F(\mathbf{y}_t | \mathbf{w}_t; \boldsymbol{\theta})$ and $H(\mathbf{y}_t | \mathbf{w}_t; \boldsymbol{\alpha}) \ \mathscr{E} \ G(\mathbf{y}_t | \mathbf{w}_t; \boldsymbol{\gamma})$ automatically, since the DGP can explain the characteristics of any model of itself, no matter how imperfect the model is. Though this relationship between the distribution functions is important conceptually, it is not of practical value since $H(\mathbf{y}_t | \mathbf{w}_t; \boldsymbol{\alpha})$ is unknown (and unknowable), and in practice the objective is to obtain congruent simple models which despite their parsimony perform as well as more general congruent models within which they are nested (the principle of parsimonious encompassing $\mathscr{E}_p$).

Now using M_1 and M_2 to denote models characterized by the densities $f(\mathbf{y}_t | \mathbf{w}_t; \boldsymbol{\theta})$ and $g(\mathbf{y}_t | \mathbf{w}_t; \boldsymbol{\gamma})$ respectively, the log–likelihood functions for models M_1 and M_2 with a finite sample size T, are given by:

$$L_f(\mathbf{y}, \boldsymbol{\theta}) = T^{-1} \Sigma_{t=1}^{T} \log f(\mathbf{y}_t | \mathbf{w}_t; \boldsymbol{\theta})$$

$$L_g(\mathbf{y}, \boldsymbol{\gamma}) = T^{-1} \Sigma_{t=1}^{T} \log g(\mathbf{y}_t | \mathbf{w}_t; \boldsymbol{\gamma})$$

Under Assumptions 1–5 the pseudo maximum likelihood estimators $\hat{\boldsymbol{\theta}}_T$ and $\hat{\gamma}_T$ exist for all T, are measurable, and solve the following maximization problems $\max_{\boldsymbol{\theta} \in \Xi} L_f(\mathbf{y},\boldsymbol{\theta})$ and $\max_{\boldsymbol{\gamma} \in \Gamma} L_g(\mathbf{y},\boldsymbol{\gamma})$ respectively (see e.g. Gourieroux, Monfort and Trognon (1983, 1984), and White (1982)). Under $H(\mathbf{y}_t|\mathbf{w}_t;\boldsymbol{\alpha})$ the estimators $\hat{\boldsymbol{\theta}}_T$ and $\hat{\gamma}_T$ have well–defined limits given in the following lemma which is proved in White (1981, Theorem 2.1 and Corollary 2.2).

Lemma 1:

Given the Assumptions 1–5 the pseudo maximum likelihood estimators $\hat{\boldsymbol{\theta}}_T$ and $\hat{\gamma}_T$ which solve $\max_{\boldsymbol{\theta} \in \Xi} L_f(\mathbf{y},\boldsymbol{\theta})$ and $\max_{\boldsymbol{\gamma} \in \Gamma} L_g(\mathbf{y},\boldsymbol{\gamma})$ converge almost surely to $\boldsymbol{\theta}(\boldsymbol{\alpha})$ and $\boldsymbol{\gamma}(\boldsymbol{\alpha})$ respectively. $\boldsymbol{\theta}(\boldsymbol{\alpha})$ and $\boldsymbol{\gamma}(\boldsymbol{\alpha})$ are the unique solutions of:

$$\max_{\boldsymbol{\theta} \in \Xi} E_w \int \log f(\mathbf{y}_t|\mathbf{w}_t;\boldsymbol{\theta})\, dH(\mathbf{y}_t|\mathbf{w}_t;\boldsymbol{\alpha})$$

$$\max_{\boldsymbol{\gamma} \in \Gamma} E_w \int \log g(\mathbf{y}_t\boldsymbol{\gamma}|\mathbf{w}_t;\boldsymbol{\gamma})\, dH(\mathbf{y}_t|\mathbf{w}_t;\boldsymbol{\alpha})$$

where $\boldsymbol{\alpha} \in A$ and $H(\mathbf{y}_t|\mathbf{w}_t;\boldsymbol{\alpha})$ is the DGP.

$\boldsymbol{\theta}(\boldsymbol{\alpha})$ and $\boldsymbol{\gamma}(\boldsymbol{\alpha})$ are the pseudo true values of $\hat{\boldsymbol{\theta}}_T$ and $\hat{\gamma}_T$ under the DGP (see e.g. Sawa (1978), Gourieroux, Monfort and Trognon (1983) and Mizon and Richard (1986)). Although the non–experimentally generated data which are typically available for econometric modelling have an unknown DGP, it is relevant to test whether an econometric model can accurately mimic the ability of the DGP to explain the behaviour of statistics of interest (e.g. parameter estimators) in other models. Indeed, this is one of the basic ideas underlying the encompassing principle (see Hendry and Richard (1982), Mizon (1984), and Mizon and Richard (1986)). In particular, it is possible to define $\boldsymbol{\gamma}^1(\boldsymbol{\theta})$ say, as the pseudo true value of $\hat{\gamma}_T$ under $F(\mathbf{y}_t|\mathbf{w}_t;\boldsymbol{\theta})$, when $\boldsymbol{\gamma}^1(\boldsymbol{\theta})$ is the unique solution to the maximization problem:

$$\max_{\boldsymbol{\gamma} \in \Gamma} E_w \int \log g(\mathbf{y}_t|\mathbf{w}_t;\boldsymbol{\gamma})\, dF(\mathbf{y}_t|\mathbf{w}_t;\boldsymbol{\theta})$$

It then follows that M_1 will accurately mimic the DGP's ability to explain the behaviour of $\hat{\gamma}_T$ (a statistic of interest in M_2) if $\boldsymbol{\gamma}^1(\boldsymbol{\theta}) = \boldsymbol{\gamma}(\boldsymbol{\theta})$. Similarly, the finite sample pseudo true value of $\hat{\gamma}_T$ under $F(\mathbf{y}_t|\mathbf{w}_t;\boldsymbol{\theta})$, $\boldsymbol{\gamma}_T^1(\boldsymbol{\theta})$ say, uniquely solves:

$$\max_{\boldsymbol{\gamma} \in \Gamma} T^{-1} \int L_g(\mathbf{y},\boldsymbol{\gamma})\, f(\mathbf{y}_t|\mathbf{w}_t;\boldsymbol{\theta})\, d\nu_y$$

For further discussion of these concepts see Gourieroux, Monfort and Trognon (1983), and for proof of the fact that $\boldsymbol{\gamma}_T^1(\boldsymbol{\theta})$ tends almost surely to $\boldsymbol{\gamma}^1(\boldsymbol{\theta})$ as $T \longrightarrow \infty$ see Burguete, Gallant and Souza (1982).

3. The Relationship Between Encompassing and M–Testing

3.1 Encompassing Hypotheses and Test Statistics

Under assumption 5 the two statistical models M_1 and M_2 are characterized by the density functions $f(\mathbf{y}_t|\mathbf{w}_t;\boldsymbol{\theta})$ and $g(\mathbf{y}_t|\mathbf{w}_t;\boldsymbol{\gamma})$ respectively. The following definitions summarize the encompassing relationships between M_1 and M_2 as discussed by Hendry and Richard (1982, 1989), Mizon (1984), and Mizon and Richard (1986).

Definition 1: Parametric Encompassing Hypotheses

Given Assumptions 1–5:

(a) $M_1 \mathscr{E} M_2$ if and only if H_ϕ: $\boldsymbol{\phi} = \boldsymbol{\gamma} - \boldsymbol{\gamma}^1(\boldsymbol{\theta}) = 0$.

(b) $M_1 \mathscr{E}(\mathbf{c})\ M_2$ if and only if H_{ϕ_c}: $\boldsymbol{\phi}_c = \mathbf{c}(\boldsymbol{\gamma}) - \mathbf{c}(\boldsymbol{\gamma}^1(\boldsymbol{\theta})) = 0$ when $\mathbf{c}(\boldsymbol{\gamma})$ is a non–random function of the parameters $\boldsymbol{\gamma}$ of M_2.

(c) $M_1 \mathscr{E}(\mathbf{b})\ M_2$ if and only if H_{ϕ_b}: $\boldsymbol{\phi}_b = E_0[\mathbf{b}(\mathbf{y},\hat{\boldsymbol{\gamma}}) - \mathbf{b}^1(\hat{\boldsymbol{\theta}})] = 0$

when $\mathbf{b}(\mathbf{y},\hat{\boldsymbol{\gamma}})$ is a general function of the data and $\hat{\boldsymbol{\gamma}}$, $\mathbf{b}^1(\boldsymbol{\theta}) = E_1[\mathbf{b}(\mathbf{y},\hat{\boldsymbol{\gamma}})]$.

H_ϕ is the hypothesis that M_1 has the property of complete parametric encompassing over M_2, $\boldsymbol{\gamma}$ being a complete parameterization of M_2. H_{ϕ_c} is the hypothesis that M_1 encompasses M_2 with respect to some deterministic function $\mathbf{c}(.)$ of $\boldsymbol{\gamma}$, an example of which is variance encompassing when M_1 and M_2 are the linear regression models. Clearly H_ϕ is sufficient for H_{ϕ_c}, whereas H_{ϕ_c} is necessary but not sufficient for H_ϕ. Hence the criterion of variance dominance, which is a particular example of H_{ϕ_b} when $\mathbf{c}(\boldsymbol{\gamma})$ is the variance (or generalized variance in a multivariate context) of M_2, is seen to be a necessary but not sufficient condition for $M_1 \mathscr{E} M_2$. H_{ϕ_b} is a more general hypothesis concerning the encompassing relationship between M_1 and M_2 which is neither necessary nor sufficient for either H_ϕ or H_{ϕ_c} in general. A well known example of H_{ϕ_b} is the implicit null hypothesis of the Cox generalized likelihood ratio test statistic, for which $\mathbf{b}(\mathbf{y},\hat{\boldsymbol{\gamma}}) = L_f(\mathbf{y},\hat{\boldsymbol{\theta}}) - L_g(\mathbf{y},\hat{\boldsymbol{\gamma}})$ – see Cox (1961,1962). Mizon (1984) and Mizon and Richard (1986) provide more details and discussion of these encompassing hypotheses, as well as showing that each of them is testable using the results summarized in the following lemma.

Lemma 2.

(a) Under H_ϕ the statistic $\hat{\boldsymbol{\phi}}_T = \hat{\boldsymbol{\gamma}}_T - \boldsymbol{\gamma}^1_T(\hat{\boldsymbol{\theta}})$ is $o_p(1)$, and $\sqrt{T}\,\hat{\boldsymbol{\phi}}_T \xrightarrow[M_1]{d} N(0,V(\hat{\boldsymbol{\phi}}_T))$ so that

$$\eta_\phi = T\hat{\boldsymbol{\phi}}_T{}'V(\hat{\boldsymbol{\phi}}_T)^\dagger\hat{\boldsymbol{\phi}}_T \xrightarrow[M_1]{d} \chi^2(r)$$

when $V(\hat{\boldsymbol{\phi}}_T)$ has rank r and $V(\hat{\boldsymbol{\phi}}_T)^\dagger$ is a generalized inverse of $V(\hat{\boldsymbol{\phi}}_T)$,

$$V(\hat{\boldsymbol{\phi}}_T) = V(\hat{\boldsymbol{\gamma}}_T) - V(\boldsymbol{\gamma}^1_T(\hat{\boldsymbol{\theta}})) = V(\hat{\boldsymbol{\gamma}}_T) - D(\boldsymbol{\theta})V(\hat{\boldsymbol{\theta}}_T)D(\boldsymbol{\theta})'$$

$$V(\hat{\boldsymbol{\gamma}}_T) = \{E_2\,[-T^{-1}\,\partial^2 L_g(\mathbf{y},\boldsymbol{\gamma})/\partial\boldsymbol{\gamma}\partial\boldsymbol{\gamma}']\}^{-1}$$

$$V(\hat{\boldsymbol{\theta}}_T) = \{E_1\,[-T^{-1}\,\partial^2 L_f(\mathbf{y},\boldsymbol{\theta})/\partial\boldsymbol{\theta}\partial\boldsymbol{\theta}']\}^{-1}$$

and

$$D(\boldsymbol{\theta}) = \partial\boldsymbol{\gamma}^1(\boldsymbol{\theta})/\partial\boldsymbol{\theta}'.$$

(b) Under H_{ϕ_c} the statistic $\hat{\boldsymbol{\phi}}_{c_T} = \mathbf{c}(\hat{\boldsymbol{\gamma}}_T) - \mathbf{c}(\boldsymbol{\gamma}^1(\hat{\boldsymbol{\theta}}_T))$ is $o_p(1)$, and

$$\sqrt{T}\,\hat{\boldsymbol{\phi}}_{c_T} \xrightarrow[M_1]{d} N(0,V(\hat{\boldsymbol{\phi}}_{c_T}))$$

so that

$$\eta_{\phi_c} = T\hat{\boldsymbol{\phi}}_{c_T}{}'V(\hat{\boldsymbol{\phi}}_{c_T})^\dagger\hat{\boldsymbol{\phi}}_{c_T} \xrightarrow[M_1]{d} \chi^2(r_c)$$

when $V(\hat{\boldsymbol{\phi}}_{c_T})$ has rank r_c and $V(\hat{\boldsymbol{\phi}}_{c_T})^\dagger$ is a generalized inverse of $V(\hat{\boldsymbol{\phi}}_{c_T})$, and $V(\hat{\boldsymbol{\phi}}_{c_T}) = CV(\hat{\boldsymbol{\phi}}_T)C'$ when $C = \partial\mathbf{c}/\partial\boldsymbol{\gamma}$.

(c) Under H_{ϕ_b} the statistic $\hat{\boldsymbol{\phi}}_{b_T} = [\mathbf{b}(\mathbf{y},\hat{\gamma}) - \mathbf{b}^1(\hat{\boldsymbol{\theta}})]$ is $o_p(1)$, and

$$\sqrt{T}\,\hat{\boldsymbol{\phi}}_{b_T} \xrightarrow[M_1]{d} N(0,V(\hat{\boldsymbol{\phi}}_{b_T}))$$

so that

$$\eta_{\phi_b} = T\hat{\boldsymbol{\phi}}_{b_T}{}'V(\hat{\boldsymbol{\phi}}_{b_T})^{\dagger}\hat{\boldsymbol{\phi}}_{b_T} \xrightarrow[M_1]{d} \chi^2(r_b)$$

when $V(\hat{\boldsymbol{\phi}}_{b_T})$ has rank r_b and $V(\hat{\boldsymbol{\phi}}_{b_T})^{\dagger}$ is a generalized inverse of $V(\hat{\boldsymbol{\phi}}_{b_T})$, and $V(\hat{\boldsymbol{\phi}}_{b_T}) = BV(\hat{\boldsymbol{\phi}}_T)B'$ when $B = \underset{M_1}{\mathrm{plim}}(\partial\mathbf{b}/\partial\boldsymbol{\gamma}')$[1].

Definition 1 and Lemma 2 are drawn from results in Mizon (1984), and Mizon and Richard (1986). Slightly narrower definitions are suggested by White (1989). However, as pointed out in each of these references, with appropriate choices of $\mathbf{c}(\gamma)$ and $\mathbf{b}(\mathbf{y},\gamma)$ virtually all tests of specification or misspecification can be obtained from Lemma 2, and so the encompassing principle can be used as a generator of test statistics. This has the advantages of: providing an encompassing interpretation for each test statistic, thus enabling an assessment to be made of the relative merits of alternative test statistics available for the evaluation of particular models; and providing a framework within which to implement tests of the relative merits of alternative models.

3.2 Conditional Moments and the M–Testing Principle.

The M–testing principle as a general framework for generating specification tests was proposed by Newey (1985) and Tauchen (1985), and discussed in detail in White (1989). Primary interest in this approach lies in "obtaining good estimates of the vector of parameters $\boldsymbol{\theta}_0$" (Newey (1985, p.1047)), when $\boldsymbol{\theta}_0$ is the population parameter vector for the model M_1, and so $\boldsymbol{\theta}_0 = \boldsymbol{\theta}(\boldsymbol{\alpha})$ in the notation of this paper. The essence of the M–testing principle is that when M_1 is correctly specified there exist s continuously differentiable, measurable, and integrable functions in the s×1 vector $\mathbf{m}_t(.)$ which satisfy the conditional moment hypothesis:

$$H_m:\ E_w\, E_1[\mathbf{m}_t(\mathbf{y}_t\,|\,\mathbf{w}_t,\boldsymbol{\theta}_0)] = 0 \qquad t = 1,2,....$$

H_m then provides a basis for specification tests of M_1. In particular, under the regularity conditions given by Assumptions 1–5, the statistic:

$$\mathbf{m}(\hat{\boldsymbol{\theta}}_T) = T^{-1}\Sigma_{t=1}^{T}\mathbf{m}_t(\mathbf{y}_t\,|\,\mathbf{w}_t,\hat{\boldsymbol{\theta}}_T) \xrightarrow[M_1]{p} 0,$$

and:

$$\sqrt{T}\,\mathbf{m}(\hat{\boldsymbol{\theta}}_T) \xrightarrow[M_1]{d} N(0,\ V(\mathbf{m}))$$

so that:

$$\mu_{\theta} = T\,\mathbf{m}(\hat{\boldsymbol{\theta}}_T)'V(\mathbf{m})^{\dagger}\mathbf{m}(\hat{\boldsymbol{\theta}}_T) \xrightarrow[M_1]{d} \chi^2(q)$$

when $\mathrm{rank}(V(\mathbf{m})) = q \leq s$.

Newey (1985), Tauchen (1985) and White (1989) provide a more detailed and general derivation of conditional moment test statistics, by for example considering GMM estimators of $\boldsymbol{\theta}$. However, the important point for present purposes is that it has been shown (see e.g. Newey (1985), Pagan (1989), Tauchen (1985), and White (1989)) that most well–known test statistics can be cast in this M–testing framework by choosing appropriate functional forms of $\mathbf{m}_t(.)$. In particular, White (1989) has shown that by choosing $\mathbf{m}(\hat{\boldsymbol{\theta}}_T) = \hat{\boldsymbol{\phi}}_T = (\hat{\boldsymbol{\gamma}}_T - \boldsymbol{\gamma}_T^1(\hat{\boldsymbol{\theta}}))$ the complete parametric encompassing test

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[1]Assuming that ϕ_* of Theorem 2 in Mizon and Richard (1986) is $o_p(1/\sqrt{T})$. Otherwise a modification along the lines indicated in their Theorem 2 is necessary.

statistic η_{ϕ} for comparing M_1 with M_2 via the hypothesis H_{ϕ}, can be generated as an M–test. Indeed, noting that the encompassing test statistics are generated by finding an encompassing contrast (e.g. $\hat{\phi}_T$, or $\hat{\phi}_{c_T}$, or $\hat{\phi}_{b_T}$) which has zero expectation, and known distribution, under the null (H_{ϕ}, or H_{ϕ_c}, or H_{ϕ_b} respectively), the commonalities between $\hat{\phi}_{b_T}$ of $\mathscr{E}$–tests and $\mathbf{m}(\hat{\theta}_T)$ of the M–test are clear. Further, as generators of test statistics, both create quadratic forms η_{ϕ} and μ_{θ} involving respectively the contrast $\hat{\phi}_{b_T}$ and the sample conditional moment $\mathbf{m}(\hat{\theta}_T)$ (and their respective limiting covariance matrices), which have limiting χ^2 distributions – central under the null, non–central under the alternative.

In order to show that there is an equivalence between the $\mathscr{E}$–testing framework and that of M–testing as generators of test statistics it is necessary to show that any $\mathscr{E}$–test statistic η_{ϕ} can be generated as an M–test statistic, and vice versa. White (1989) has established the former. The next section proves that for every conditional moment hypothesis H_m and its associated test statistic μ_{θ}, there is a corresponding parsimonious encompassing hypothesis and test statistic η_{ϕ}. The equivalence between the testing frameworks is then established by showing the links between encompassing and parsimonious encompassing.

4. Encompassing Within the Exponentially Extended Family of Distributions.

Consider the k dimensional vector of continuously differentiable, measurable, and integrable functions $\boldsymbol{\psi}_t(\mathbf{y}_t|\mathbf{w}_t,\boldsymbol{\theta})$ for which (unlike $\mathbf{m}_t(\mathbf{y}_t|\mathbf{w}_t,\boldsymbol{\theta})$) the conditional moment condition $E_w\, E_1[\boldsymbol{\psi}_t(\mathbf{y}_t|\mathbf{w}_t,\boldsymbol{\theta})] = 0$ does not necessarily hold. These functions together with M_1 as defined in Assumption 5 can be used to construct the exponentially extended class of models M^*:

$$M^*: \{f^*(\mathbf{y}_t|\mathbf{w}_t;\boldsymbol{\theta},\boldsymbol{\lambda}) = \exp[\boldsymbol{\lambda}'\boldsymbol{\psi}_t(\mathbf{y}_t|\mathbf{w}_t,\boldsymbol{\theta})]\, f(\mathbf{y}_t|\mathbf{w}_t;\boldsymbol{\theta})/Q_t\ ;\ \boldsymbol{\theta}\in\Xi,\ \boldsymbol{\lambda}\in\Lambda\subset\mathbb{R}^k\}$$

when $Q_t = \int \exp[\boldsymbol{\lambda}'\boldsymbol{\psi}_t(\mathbf{y}_t|\mathbf{w}_t,\boldsymbol{\theta})]\, f(\mathbf{y}_t|\mathbf{w}_t;\boldsymbol{\theta})\, d\nu_y$.

For the special case in which $\boldsymbol{\psi}_t(\mathbf{y}_t|\mathbf{w}_t,\boldsymbol{\theta}) = \mathbf{y}_t$, $f^*(\mathbf{y}_t|\mathbf{w}_t;\boldsymbol{\theta},\boldsymbol{\lambda})$ is called the *exponential tilt* of $f(\mathbf{y}_t|\mathbf{w}_t,\boldsymbol{\theta})$, and can be considered as a saddlepoint approximation to $f(\mathbf{y}_t|\mathbf{w}_t,\boldsymbol{\theta})$ – see Barndorff–Nielsen and Cox (1989).

Manifestly, $f^*(\mathbf{y}_t|\mathbf{w}_t;\boldsymbol{\theta},\boldsymbol{\lambda}) = f(\mathbf{y}_t|\mathbf{w}_t;\boldsymbol{\theta})$ for $\boldsymbol{\lambda} = 0$, and in this sense $f(\mathbf{y}_t|\mathbf{w}_t;\boldsymbol{\theta})$ is nested within $f^*(\mathbf{y}_t|\mathbf{w}_t;\boldsymbol{\theta},\boldsymbol{\lambda})$, so that $M_1 \subset M^*$. Hence M^* automatically encompasses M_1, and so from a modelling perspective it is much more interesting to test whether M_1 is a valid reduction of M^*, that is whether $M_1\ \mathscr{E}_p\ M^*$. Letting $\tilde{\boldsymbol{\lambda}}_T$ be the maximum likelihood estimator of $\boldsymbol{\lambda}$ in M^*, and $\boldsymbol{\lambda}^* = E_0(\tilde{\boldsymbol{\lambda}}_T)$, the hypothesis that $M_1\ \mathscr{E}_p\ M^*$ is equivalent to H_{λ}: $\boldsymbol{\lambda}^* = 0$. It is now possible to use the exponentially extended family of densities in M^* to (i) link $\mathscr{E}_p$ and the Cox generalized likelihood ratio test, (ii) to relate the properties of $\mathscr{E}$ and $\mathscr{E}_p$ for dynamic nonlinear models, and (iii) to establish an equivalence between between $\mathscr{E}_p$–tests and M–tests.

Theorem

Under the Assumptions 1–5 and in the context of M^* the hypothesis H_λ: $\boldsymbol{\lambda}^* = 0$ is equivalent to the hypothesis H_ψ: $\boldsymbol{\psi}_0 - \boldsymbol{\psi}_1 = 0$ when $\boldsymbol{\psi}_0 = E_0[\boldsymbol{\psi}_t(\mathbf{y}_t|\mathbf{w}_t,\boldsymbol{\theta}_0)]$ and $\boldsymbol{\psi}_1 = E_1[\boldsymbol{\psi}_t(\mathbf{y}_t|\mathbf{w}_t,\boldsymbol{\theta}_0)]$ with $\boldsymbol{\theta}_0 = \boldsymbol{\theta}(\boldsymbol{\alpha})$.

Proof. The log–likelihood function L^* for model M^*, which under the regularity conditions given above has unique maximizers $\tilde{\boldsymbol{\theta}}_T$ and $\tilde{\boldsymbol{\lambda}}_T$, is given by:

$$L^* = L_f + \Sigma_{t=1}^T \boldsymbol{\lambda}' \boldsymbol{\psi}_t(\mathbf{y}_t|\mathbf{w}_t,\boldsymbol{\theta}) - \Sigma_{t=1}^T \log Q_t$$

so that:

$$\frac{\partial L^*}{\partial \boldsymbol{\lambda}} = \Sigma_{t=1}^T \boldsymbol{\psi}_t(\mathbf{y}_t|\mathbf{w}_t,\boldsymbol{\theta}) - \Sigma_{t=1}^T \{\textstyle\int \boldsymbol{\psi}_t(\mathbf{y}_t|\mathbf{w}_t,\boldsymbol{\theta})\exp[\boldsymbol{\lambda}' \boldsymbol{\psi}_t(\mathbf{y}_t|\mathbf{w}_t,\boldsymbol{\theta})]f(\mathbf{y}_t|\mathbf{w}_t,\boldsymbol{\theta})d\nu_y\}/Q_t$$
$$= \Sigma_{t=1}^T \boldsymbol{\psi}_t(\mathbf{y}_t|\mathbf{w}_t,\boldsymbol{\theta}) - \Sigma_{t=1}^T E^*[\boldsymbol{\psi}_t(\mathbf{y}_t|\mathbf{w}_t,\boldsymbol{\theta})]$$

when E^* denotes the expectation operator with respect to M^*. Hence the maximum likelihood estimators, $\tilde{\boldsymbol{\lambda}}_T$ of $\boldsymbol{\lambda}$, and $\tilde{\boldsymbol{\theta}}_T$ of $\boldsymbol{\theta}$, satisfy the equation:

$$\Sigma_{t=1}^T \boldsymbol{\psi}_t(\mathbf{y}_t|\mathbf{w}_t,\tilde{\boldsymbol{\theta}}_T) - \Sigma_{t=1}^T E^*[\boldsymbol{\psi}_t(\mathbf{y}_t|\mathbf{w}_t,\boldsymbol{\theta})]_{\tilde{\boldsymbol{\lambda}}_T,\tilde{\boldsymbol{\theta}}_T} = 0$$

and so letting:

$$\hat{\boldsymbol{\psi}}^* = T^{-1}\Sigma_{t=1}^T E^*[\boldsymbol{\psi}_t(\mathbf{y}_t|\mathbf{w}_t,\boldsymbol{\theta})]_{\tilde{\boldsymbol{\lambda}}_T,\tilde{\boldsymbol{\theta}}_T}$$

it follows that:

$$\text{plim } \hat{\boldsymbol{\psi}}^* = E^*[\boldsymbol{\psi}_t(\mathbf{y}_t|\mathbf{w}_t,\boldsymbol{\theta})]_{\boldsymbol{\lambda}_*,\boldsymbol{\theta}_*} = E_0[\boldsymbol{\psi}_t(\mathbf{y}_t|\mathbf{w}_t,\boldsymbol{\theta}_*)]$$

where $\boldsymbol{\lambda}_* = \text{plim } \tilde{\boldsymbol{\lambda}}_T$, and $\boldsymbol{\theta}_* = \text{plim } \tilde{\boldsymbol{\theta}}_T$. However, if $\boldsymbol{\lambda}_* = 0$ (so that $M_1 = M^*$) then

$$E_0[\boldsymbol{\psi}_t(\mathbf{y}_t|\mathbf{w}_t,\boldsymbol{\theta}_*)] = E_0[\boldsymbol{\psi}_t(\mathbf{y}_t|\mathbf{w}_t,\boldsymbol{\theta}_0)] = E_1[\boldsymbol{\psi}_t(\mathbf{y}_t|\mathbf{w}_t,\boldsymbol{\theta}_0)]$$

and hence,

$$\text{plim } \hat{\boldsymbol{\psi}}^* = E_1[\boldsymbol{\psi}_t(\mathbf{y}_t|\mathbf{w}_t,\boldsymbol{\theta}_0)].$$

Conversely, if plim $\hat{\boldsymbol{\psi}}^* = E_1[\boldsymbol{\psi}_t(\mathbf{y}_t|\mathbf{w}_t,\boldsymbol{\theta}_0)]$ then,

$$\{\textstyle\int \boldsymbol{\psi}_t(\mathbf{y}_t|\mathbf{w}_t,\boldsymbol{\theta})\exp[\boldsymbol{\lambda}' \boldsymbol{\psi}_t(\mathbf{y}_t|\mathbf{w}_t,\boldsymbol{\theta})]f(\mathbf{y}_t|\mathbf{w}_t,\boldsymbol{\theta})d\nu_y/Q_t\}_{\boldsymbol{\lambda}_*,\boldsymbol{\theta}_*} = \textstyle\int \boldsymbol{\psi}_t(\mathbf{y}_t|\mathbf{w}_t,\boldsymbol{\theta})f(\mathbf{y}_t|\mathbf{w}_t,\boldsymbol{\theta}_0)d\nu_y$$

which, since $\boldsymbol{\theta}_*$ and $\boldsymbol{\lambda}_*$ are the unique maximizers of $E_w \int \log f^*(\mathbf{y}_t|\mathbf{w}_t;\boldsymbol{\theta},\boldsymbol{\lambda})\, dH(\mathbf{y}_t|\mathbf{w}_t;\boldsymbol{\alpha})$, where $\boldsymbol{\alpha} \in A$ and $H(\mathbf{y}_t|\mathbf{w}_t;\boldsymbol{\alpha})$ is the DGP, implies that $\boldsymbol{\lambda}_* = 0$ and $\boldsymbol{\theta}_* = \boldsymbol{\theta}_0$ □

Considering the special case in which $\psi_t(\mathbf{y}_t|\mathbf{w}_t,\boldsymbol{\theta})$ is chosen to be $\mathbf{m}_t(\mathbf{y}_t|\mathbf{w}_t,\boldsymbol{\theta})$ of the conditional moment test (so that $\boldsymbol{\psi}_1 = E_1[\boldsymbol{\psi}_t(\mathbf{y}_t|\mathbf{w}_t,\boldsymbol{\theta}_0)] = E_1[\mathbf{m}_t(\mathbf{y}_t|\mathbf{w}_t,\boldsymbol{\theta}_0)] = 0$ under the null H_m)) yields the following corollary (the proof of which is straightforward) relating parsimonious encompassing and M–testing.

Corollary
Under the Assumptions 1–5 the conditional moment hypothesis H_m: $E_1[\mathbf{m}_t(\mathbf{y}_t|\mathbf{w}_t,\boldsymbol{\theta}_0)] = 0$ is equivalent to the hypothesis H_λ: $\boldsymbol{\lambda}^* = 0$, which in turn is equivalent to the hypothesis that $M_1\ \mathscr{E}_p\ M^*$.

This Corollary shows that in the context of departures from M_1 in the direction of $E_1[\mathbf{m}_t(\mathbf{y}_t|\mathbf{w}_t,\boldsymbol{\theta}_0)] \neq 0$, as defined by the exponentially extended family of densities:

$$M^*:\ \{f^*(\mathbf{y}_t|\mathbf{w}_t;\boldsymbol{\theta},\boldsymbol{\lambda}) = \exp[\boldsymbol{\lambda}'\mathbf{m}_t(\mathbf{y}_t|\mathbf{w}_t,\boldsymbol{\theta})]\ f(\mathbf{y}_t|\mathbf{w}_t;\boldsymbol{\theta})/Q_t\ ;\ \boldsymbol{\theta}\in\Xi,\ \boldsymbol{\lambda}\in\Lambda\subset\mathbb{R}^k\}$$

when $Q_t = \int \exp[\boldsymbol{\lambda}'\mathbf{m}_t(\mathbf{y}_t|\mathbf{w}_t,\boldsymbol{\theta})]\ f(\mathbf{y}_t|\mathbf{w}_t;\boldsymbol{\theta})\ d\nu_y$, the principles of parsimonious encompassing and conditional moment testing yield equivalent hypothesis tests of the specification of M_1. That H_λ: $\boldsymbol{\lambda}^* = 0$ is equivalent to the hypothesis that $M_1\ \mathscr{E}_p\ M^*$ is obvious. The import of the Corollary is that for any conditional moment hypothesis H_m: $E_1[\mathbf{m}_t(\mathbf{y}_t|\mathbf{w}_t,\boldsymbol{\theta}_0)] = 0$ there is a parsimonious encompassing hypothesis $(H_\lambda\text{: } \boldsymbol{\lambda}^* = 0) \equiv (M_1\ \mathscr{E}_p\ M^*)$. Hence provided that there is an equivalence between encompassing $\mathscr{E}$ and parsimonious encompassing $\mathscr{E}_p$, the encompassing principle and the M–testing framework are each capable of generating test statistics which have implicit null hypotheses that are equivalent. The equivalence of the two approaches to generating test statistics is then established by noting that Hendry and Richard (1989) showed the equivalence of encompassing and parsimonious encompassing in the following sense: H_ϕ: $M_1\ \mathscr{E}\ M_2$ if and only if $M_1\ \mathscr{E}_p\ M_c$ when $M_1 \subset M_c$ and $M_2 \subset M_c$.

Although as generators of test statistics the encompassing principle and the M–testing framework are equivalent, this does not imply that either of them is redundant. In particular, one of the potential attractions of the M–testing approach, mentioned by Pagan (1989), is that computer programs which are available to solve estimator generating equations of the form $T^{-1}\sum_{t=1}^{T} \mathbf{d}_t(\mathbf{y}_t|\mathbf{w}_t,\hat{\boldsymbol{\theta}}_T) = 0$ for $\hat{\boldsymbol{\theta}}_T$, (e.g. for maximum likelihood, instrumental variables, or GMM estimators), can be used to solve:

$$T^{-1}\sum_{t=1}^{T} \varphi_t(\mathbf{y}_t|\mathbf{w}_t,\hat{\boldsymbol{\theta}}_T,\hat{\tau}_T) = 0 \quad \text{for } \hat{\boldsymbol{\theta}}_T \text{ and } \hat{\tau}_T$$

when

$$\varphi_t(\mathbf{y}_t|\mathbf{w}_t,\hat{\boldsymbol{\theta}}_T,\hat{\tau})' = [\mathbf{d}_t(\mathbf{y}_t|\mathbf{w}_t,\hat{\boldsymbol{\theta}}_T)',\ (\mathbf{m}_t(\mathbf{y}_t|\mathbf{w}_t,\hat{\boldsymbol{\theta}}_T) - \hat{\tau}_T)']$$

which will yield $\hat{\tau}_T = \mathbf{m}(\hat{\boldsymbol{\theta}}_T)$ together with the associated asymptotic covariance estimator. This potential advantage though will only be realized if the statistic $\mathbf{m}(\hat{\boldsymbol{\theta}}_T) = T^{-1}\sum_{t=1}^{T} \mathbf{m}_t(\mathbf{y}_t|\mathbf{w}_t,\hat{\boldsymbol{\theta}}_T)$ is readily calculable, which it is for most of the published applications of M–testing, but it is not in all cases. Indeed, a case in which $\mathbf{m}(\hat{\boldsymbol{\theta}}_T)$ will not be calculated easily occurs when it is desired to check the adequacy of the specification of M_1 in the direction of the alternative model M_2 so that $\mathbf{m}(\hat{\boldsymbol{\theta}}_T) = \hat{\boldsymbol{\phi}}_T = \hat{\gamma}_T - \gamma_T^1(\hat{\boldsymbol{\theta}}_T)$ but the pseudo true value $\gamma_T^1(\hat{\boldsymbol{\theta}}_T)$ is not derivable analytically as for example when M_1 and M_2 are nonlinear dynamic models. Although this particular illustration of difficulties in calculating $\mathbf{m}(\hat{\boldsymbol{\theta}}_T)$ is associated with an encompassing hypothesis the problem is not peculiar to encompassing. Moreover, one of the attractions of encompassing hypotheses is the use of specific alternatives to the null model M_1, which means that even when analytical pseudo true values are not available it is possible to use Monte Carlo simulation to estimate them – see Gourieroux and Monfort (1992), Lu and Mizon (1990), and Pesaran and Pesaran (1992).

An important difference between the two approaches concerns the motivation for using a test statistic. In the encompassing approach the purpose is usually to compare the relative merits of alternative models such as M_1 and M_2, and hence parsimonious encompassing will be with respect to an M_c which is a completion of M_1 and M_2, so that $M_1 \subset M_c$ and $M_2 \subset M_c$ (not necessarily $\mathbf{M}_c = M^*$ with $\psi_t(\mathbf{y}_t|\mathbf{w}_t,\boldsymbol{\theta}) = \log[f(\mathbf{y}_t|\mathbf{w}_t,\boldsymbol{\theta})/g(\mathbf{y}_t|\mathbf{w}_t,\boldsymbol{\gamma})]$ – a case which is considered in the next section), which is also congruent. In the M–testing approach, on the other hand, primary interest lies in obtaining good estimates of the parameters of M_1, so that there is often no alternative model M_2 of interest, only a conditional moment hypothesis H_m embodying a diagnostic check of M_1. In this latter case an appropriate completing model M_c for the corresponding parsimonious encompassing test is M^* with $\psi_t(\mathbf{y}_t|\mathbf{w}_t,\boldsymbol{\theta}) = \mathbf{m}_t(\mathbf{y}_t|\mathbf{w}_t,\boldsymbol{\theta})$. Therefore, it is in the choice (perhaps implicit) of the completing model M_c to support the parsimonious encompassing interpretation of each approach, that the difference in motivation and emphasis of the approaches lies. Hence although any particular hypothesis could be tested using either approach, it will usually be the case that encompassing tests will be used for specification tests (i.e. comparison of alternative models), whereas M–tests will be used for diagnostic checks or tests of misspecification of the single model of interest M_1.

5. Applications of Encompassing Within the Exponentially Extended Family

In this section attention is confined to completing models M_c within the exponentially extended family of densities M^*. The first application considers the comparison of two nonlinear non–nested models, and provides a parsimonious encompassing interpretation of Cox's generalized likelihood ratio test relative to the M_n^* class of densities analyzed by Atkinson (1970). The second application illustrates how the well known choice of regressors problem in the Gaussian case fits naturally into the parsimonious encompassing framework provided by the completing model M^*.

5.1 Testing Nonlinear Non–nested Hypotheses

Consider the two generic models M_1 and M_2 as defined in Assumption 5, when neither model can be obtained by imposing restrictions on the parameter space of the other, so that they are non–nested. To test the hypothesis M_1 against the non–nested alternative M_2, construct the general model M_n^*:

$$M_n^*: \{f^*(y_t|\mathbf{w}_t,\boldsymbol{\theta},\boldsymbol{\gamma},\zeta) = f(y_t|\mathbf{w}_t,\boldsymbol{\theta})^{1-\zeta}g(y_t|\mathbf{w}_t,\boldsymbol{\gamma})^{\zeta}/Q_t, \qquad \zeta \in [0, 1]\}$$

where

$$Q_t = \int f(y_t|\mathbf{w}_t,\boldsymbol{\theta})^{1-\zeta}g(y_t|\mathbf{w}_t,\boldsymbol{\gamma})^{\zeta}\, d\nu_y.$$

Note that the density function $f^*(y_t|\mathbf{w}_t,\boldsymbol{\theta},\boldsymbol{\gamma},\zeta)$ can be re–written as:

$$f^*(y_t|\mathbf{w}_t,\boldsymbol{\theta},\boldsymbol{\gamma},\zeta) = \exp\{-\zeta(\log[f(y_t|\mathbf{w}_t,\boldsymbol{\theta})/g(y_t|\mathbf{w}_t,\boldsymbol{\gamma})])\}\cdot f(y_t|\mathbf{w}_t,\boldsymbol{\theta})/Q_t$$

Let $\lambda = -\zeta$, and $\psi_t(y_t|\mathbf{w}_t,\boldsymbol{\theta}) = \log[f(y_t|\mathbf{w}_t,\boldsymbol{\theta})/g(y_t|\mathbf{w}_t,\boldsymbol{\gamma})]$, then $f^*(y_t|\mathbf{w}_t,\boldsymbol{\theta},\boldsymbol{\gamma},\zeta)$ belongs to the exponential family generated by λ and ψ_t. From the Theorem in section 4 it follows that M_1 parsimoniously encompasses M_n^* if and only if:

$$T^{-1}\Sigma_{t=1}^{T}\log[f(y_t|\mathbf{w}_t,\boldsymbol{\theta})/g(y_t|\mathbf{w}_t,\boldsymbol{\gamma})] - E_1\log[f(y_t|\mathbf{w}_t,\boldsymbol{\theta})/g(y_t|\mathbf{w}_t,\boldsymbol{\gamma})] \xrightarrow{p} 0$$

The hypothesis that this holds is the implicit null hypothesis of the well known generalized likelihood ratio test statistic of Cox (1961, 1962) for testing M_1 against the non–nested alternative M_2. This establishes that for nonlinear non–nested models M_1

and M_2 the encompassing hypothesis H_{ϕ_ψ} : $M_1\ \mathcal{E}(\psi_t)\ M_2$, and the parsimonious encompassing hypothesis $M_1\ \mathcal{E}_p\ M_n^*$ are equivalent. Noting that the hypothesis $M_1\ \mathcal{E}_p\ M_n^*$ is the same as the hypothesis H_ζ: $\zeta = 0$, and that the latter can be tested via a Lagrange multiplier or score test statistic, also casts the test proposed by Atkinson (1970) in the parsimonious encompassing framework. The statistics for implementing a test of H_ζ (or its equivalents) will have a limiting central χ^2 distribution with one degree of freedom.

5.2. Linear Regression Models

Consider two non–nested linear regression models given by:

$$M_1:\ y_t = \mathbf{x}_t'\boldsymbol{\beta} + u_t \qquad\qquad u_t \sim N(0, \sigma^2)$$

$$M_2:\ y_t = \mathbf{z}_t'\boldsymbol{\gamma} + v_t \qquad\qquad v_t \sim N(0, \sigma^2)$$

The hypothesis that M_1 completely parametrically encompasses M_2 is equivalent to the hypothesis that $M_1\ \mathcal{E}_p\ M_c$ when M_c: $y_t = \mathbf{x}_t'\mathbf{b} + \mathbf{z}_t'\mathbf{c} + \epsilon_t$ i.e. to the hypothesis H_c: $\mathbf{c} = 0$ – see Hendry and Richard (1989). Casting this problem into the framework of the exponentially extended family of densities reveals the relationship between M^* and M_c in this case.

The two density functions of models M_1 and M_2 are given by:

$$f(y_t|\mathbf{w}_t,\boldsymbol{\beta}) = (1/\sqrt{2\pi\sigma^2})\exp((y_t-\mathbf{x}_t'\boldsymbol{\beta})^2/2\sigma^2)$$
$$g(y_t|\mathbf{w}_t,\boldsymbol{\gamma}) = (1/\sqrt{2\pi\sigma^2})\exp((y_t-\mathbf{z}_t'\boldsymbol{\gamma})^2/2\sigma^2)$$

when $\mathbf{w}_t' = (\mathbf{x}_t',\mathbf{z}_t')$ and the density function of the completing model M^* is:

$$f^*(y_t|\mathbf{w}_t,\boldsymbol{\beta},\boldsymbol{\gamma},\zeta) = (1/\sqrt{2\pi\sigma^2})\exp\{[(1-\zeta)(y_t-\mathbf{x}_t'\boldsymbol{\beta})^2 + \zeta(y_t-\mathbf{z}_t'\boldsymbol{\gamma})^2]/2\sigma^2\}/Q_t$$

where

$$Q_t = \int(1/\sqrt{2\pi\sigma^2})\exp\{[(1-\zeta)(y_t-\mathbf{x}_t'\boldsymbol{\beta})^2 + \zeta(y_t-\mathbf{z}_t'\boldsymbol{\gamma})^2]/2\sigma^2\}dy.$$

From which it follows that M^* corresponds to a normal distribution with mean $E(y_t|\mathbf{w}_t) = \mathbf{x}_t'\boldsymbol{\beta}^* + \mathbf{z}_t'\boldsymbol{\gamma}^*$, when $\boldsymbol{\beta}^* = (1-\zeta)\boldsymbol{\beta}$, and $\boldsymbol{\gamma}^* = \zeta\boldsymbol{\gamma}$. The relationship between M_c and M^* is now provided by noting that $\boldsymbol{\beta}^* = \mathbf{b}$ and $\boldsymbol{\gamma}^* = \mathbf{c}$ – see Atkinson (1970, p327) who considers the case in which there is a single regressor in each of $\mathbf{x}_t$ and $\mathbf{z}_t$. The natural parameterization of M_c is in terms of $\mathbf{b}$, $\mathbf{c}$ and σ^2 for which it is clear that $M_1\ \mathcal{E}\ M_2$ is equivalent to $M_1\ \mathcal{E}_p\ M_c$, both of which are equivalent to $\mathbf{c} = 0$, so that the statistic to implement a test of this hypothesis will have a limiting central χ^2 distribution under the null with degrees of freedom equal to the dimension of $\mathbf{z}_t$. On the other hand the natural parameterization of M^* is in terms of $\boldsymbol{\beta}$, $\boldsymbol{\gamma}$, ζ and σ^2, for which the hypothesis $M_1\ \mathcal{E}_p\ M^*$ is equivalent to $\zeta = 0$, for which the appropriate test statistic will have a limiting central χ^2 distribution with one degree of freedom. Further, the choice $\boldsymbol{\psi}_t = \mathbf{m}_t$ with $T\mathbf{m}(\hat{\boldsymbol{\theta}}_T) = Z'Q_x\mathbf{y}$, with $Q_x = [I - X(X'X)^{-1}X']$, yields a form of M^* which is identical to M_c with the hypothesis H_λ: $\boldsymbol{\lambda} = 0$ identical to the hypothesis $\mathbf{c} = 0$. Finally note that it is not necessary to restrict M_1 and M_2 to have the same error variance σ^2. In particular, if in $g(y_t|\mathbf{w}_t,\boldsymbol{\gamma})$ σ^2 is replaced by τ^2 then with $\zeta_1 = \zeta/\sigma^2$, $\zeta_2 = (1-\zeta)/\tau^2$, and $\delta = \zeta_2/(\zeta_1+\zeta_2)$, it follows that $E(y_t|\mathbf{w}_t) = \mathbf{x}_t'\boldsymbol{\beta}^* + \mathbf{z}_t'\boldsymbol{\gamma}^*$ when $\boldsymbol{\beta}^* = (1-\delta)\boldsymbol{\beta}$, $\boldsymbol{\gamma}^* = \delta\boldsymbol{\gamma}$ and $V(y_t|\mathbf{w}_t) = (\zeta_1+\zeta_2)^{-1}$.

6. Conclusion

Although the motivation for the derivation of each was different, the principles of parsimonious encompassing and conditional moment testing are capable of testing equivalent hypotheses, and hence can generate equivalent test statistics for any given hypothesis. Hence given a model M_1 and a particular direction in which it is desired to test the adequacy of its specification, either principle can be used to generate an appropriate test statistic. When it is desired to test a particular model M_1 against a particular alternative specification M_2 it is likely that an encompassing test statistic will be used, though the above results show that it will be possible to use an equivalent conditional moment test statistic. When primary interest is in a particular model M_1, and the purpose of testing is to check the adequacy of its specification, then it is likely that a conditional moment test statistic will be used, even though an equivalent encompassing test statistic will be available. These statements are mirrored in the fact that encompassing is particularly associated with specification testing and general–to–specific modelling, whereas M–testing is particularly associated with misspecification testing and specific–to–general modelling.

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